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# Measuring CP-violation with a neutrino factory

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### Abstract

We discuss the prospects of observing leptonic CP-violation at a neutrino factory in the context of the standard three neutrino scenario. If the large mixing angle MSW effect turned out to account for the solar neutrino deficit, we show that observing an asymmetry between the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillation probabilities would represent an exciting experimental challenge. We determine the portion of the parameter space where an evidence could be found as a function of the intensity of the muon source and of the detector size for different baselines. We discuss the consequent requirements on a neutrino factory. We address the problems associated with asymmetries induced by the experimental apparatus and by matter effects. Finally, we emphasize the importance of measuring two CP-conjugated channels in order to precisely determine $\theta_{13}$.

## 1 Introduction

An increasing evidence for the existence of neutrino oscillations has been gathered by several experiments. The standard theoretical interpretations of the data requires the neutrino flavour eigenstates to be superpositions of three or more mass eigenstates. This involves the possibility of a violation of CP associated with the physical phases of the mixing matrix.

An evidence of CP-violation has been provided in the quark sector by different experiments, that have suggested a value of the CKM phase close to maximal. It would be extremely interesting to be able to investigate the CP-violation issue also in the lepton sector. Unfortunately, the upcoming long-baseline experiments will not help in this respect since they are not conceived for the purpose of comparing CP-conjugated transition. Note, however, that they could be affected by CP-violation in a relevant way. The oscillation probabilities they measure do in fact contain a CP-violating part. A detailed analysis [1] has shown that the latter can constitute a relevant part of the full oscillation probability

in the $\nu_e \leftrightarrow \nu_\mu$ channel and could therefore affect the measurement of the mixing angle $\theta_{13}$.

An experimental study of CP-violation in neutrino oscillations would require a further generation of experiments offering the possibility of measuring oscillation probabilities in two CP-conjugated channels. Such possibility has been made concrete by the proposal of using the very intense muon sources that are currently being designed as part of the muon collider projects to produce an high-intensity neutrino beam [2, 3, 4]. As a pure source of both neutrinos and antineutrinos with well known initial flux, such a "neutrino factory" would be an ideal framework for studying leptonic CP-violation. The possibility of exploiting a neutrino factory for a measurement of CP-violation has been first considered in [3], where the capabilities of a reference set-up have been studied in different points of the neutrino parameter space. An update of that analysis can be found in [5]. The purpose of this paper is to explore the full parameter space and to show in which part of it the detection of a CP-violating effect would be possible. The portion of the parameter space that could be covered will be determined as a function of the intensity of the muon source and of the size and efficiency of the detector for two different baselines. This will make clear which values of the factory parameters are necessary in order to cover a given portion of the parameter space. We address these issues in the context of three neutrino oscillations. Analysis of CP-violation in four neutrino oscillations can be found in [6, 1, 5, 7].

The paper is organized as follows. In Section 2 we introduce the usual notations for the neutrino masses and mixings and we describe the experimental set-up. In Section 3 we discuss the problems involved with a realistic measurement of CP-violation. In particular we describe how we cope with the asymmetry induced by the experimental apparatus and by matter effects. The systematic and statistical errors associated with the measurement are also discussed. In Section 4 we determine the optimal values of some parameters of the experiment and we present the results about its capabilities for different experimental configurations. In Section 5 we discuss the results and we conclude.

## 2 Framework

### 2.1 Masses, mixings and probabilities

In this paper we consider the possibility of measuring a violation of CP in the leptonic sector by means of a neutrino factory [2]. We address this issue in the context of three neutrino oscillations. The leptonic charged current is then given by

$$\bar{e}_i \gamma^\mu \nu_{e_i} = \bar{e}_i \gamma^\mu U_{ij} \nu_j, \tag{1}$$

where $U_{ij}$ is the $3\times 3$ unitary lepton mixing matrix and $e_i$, $\nu_i$, $i = 1\ldots 3$ are the left-handed charged lepton and neutrino mass eigenstates. By convention we order the neutrino mass eigenstates in such a way that $m_{\nu_1} < m_{\nu_2}$ and $\Delta m^2_{21} < |\Delta m^2_{31}|, |\Delta m^2_{32}|$, where we denote as usual $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$. The common sign of $\Delta m^2_{31}$ and $\Delta m^2_{32}$ is not determined at present.

We use the standard parameterization for $U$,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where possible Majorana phases are omitted because they do not enter the oscillation formulae in the safe approximation in which lepton number violating oscillations are neglected[1]. In that approximation, the $\nu_{e_i} \to \nu_{e_i}$, $\bar{\nu}_{e_i} \to \bar{\nu}_{e_j}$ oscillation probabilities in vacuum are

$$P(\nu_{e_i} \to \nu_{e_j}) = P_{CP}(\nu_{e_i} \to \nu_{e_j}) + P_{\not{CP}}(\nu_{e_i} \to \nu_{e_j}) \tag{3a}$$

$$P(\bar{\nu}_{e_i} \to \bar{\nu}_{e_j}) = P_{CP}(\nu_{e_i} \to \nu_{e_j}) - P_{\not{CP}}(\nu_{e_i} \to \nu_{e_j}), \tag{3b}$$

where

$$P_{CP}(\nu_{e_i} \to \nu_{e_j}) = \delta_{ij} - 4\,\mathrm{Re}\,J_{12}^{ji}\sin^2\Delta_{12} - 4\,\mathrm{Re}\,J_{23}^{ji}\sin^2\Delta_{23} - 4\,\mathrm{Re}\,J_{31}^{ji}\sin^2\Delta_{31}, \tag{4a}$$

$$P_{\not{CP}}(\nu_{e_i} \to \nu_{e_j}) = -8\sigma_{ij} J \sin\Delta_{12}\sin\Delta_{23}\sin\Delta_{31}, \tag{4b}$$

$J_{kh}^{e_i e_j} \equiv U_{e_i\nu_k}U^\dagger_{\nu_k e_j}U_{e_j\nu_h}U^\dagger_{\nu_h e_i}$, $\Delta_{kh} \equiv \Delta m^2_{kh}L/4E$, $\sigma_{ij} \equiv \sum_k \varepsilon_{ijk}$ and

$$8J = \cos\theta_{13}\sin(2\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23})\sin\delta. \tag{5}$$

The ranges we use for the neutrino masses and mixings are those obtained by the standard fits of the atmospheric and solar neutrino data [8, 9, 10, 11] taking into account the constraints given by the CHOOZ experiment [12]. Among the three possible standard solutions of the solar neutrino problem we only consider in detail the "large angle" one. In the "small angle" solution, as well as in the "vacuum" solution, CP-violation effects are in fact too small to be detectable, since they are suppressed either by the small value of $\theta_{12}$ (small angle solution) or by the small value of $\Delta m^2_{32}$ (vacuum solution).

### 2.2 Experimental set-up

The possibility of using the straight section of a high intensity muon storage ring as a neutrino factory has been emphasized in [2, 3]. Being a pure and high intensity source of both neutrinos and antineutrinos with well known initial flux, a neutrino factory would give the possibility of comparing the oscillation probabilities in two CP-conjugated channels: $\nu_{e_i} \to \nu_{e_j}/\bar{\nu}_{e_i} \to \bar{\nu}_{e_j}$ $(i \neq j)$. Therefore, it would be the ideal framework for studying leptonic CP-violation. Among the possible channels made available by a neutrino factory, the most suitable for studying CP-violation are $\nu_e \to \nu_\mu/\bar{\nu}_e \to \bar{\nu}_\mu$ and the T-conjugated $\nu_\mu \to \nu_e/\bar{\nu}_\mu \to \bar{\nu}_e$. In fact, in these channels the CP-violating part of the oscillation probability is not submerged by the CP-conserving part (as it is for the $\nu_\mu \leftrightarrow \nu_\tau$ channels), so that large asymmetries between the CP-conjugated channels can arise [1]. We consider here in particular the $\nu_e \to \nu_\mu/\bar{\nu}_e \to \bar{\nu}_\mu$ channels, since telling $\mu^+$ from $\mu^-$ in a large high-density detector is easier than telling $e^+$ from $e^-$ [3].

[1]We prefer to think of three Majorana neutrinos but in the mentioned approximation what follows apply to the case of three purely Dirac neutrinos as well.

The $\nu_e \to \nu_\mu$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) oscillation probability can be measured as follows: the electron neutrinos (antineutrinos) are produced by the decay of $N_{\mu^+}$ positive muons ($N_{\mu^-}$ negative muons) in the straight section of the storage ring pointing to the detector; then the $\nu_\mu$ ($\bar{\nu}_\mu$) produced by the oscillation can be detected by their charged current interactions in the detector. The number $n(\nu_\mu)$ ($n(\bar{\nu}_\mu)$) of "observed" $\nu_\mu$ ($\bar{\nu}_\mu$) is then given by

$$n(\nu_\mu) = N_{\mu^+} N_{\mathrm{kT}} \frac{10^9 N_A}{m_\mu^2 \pi} \frac{E_\mu^3}{L^2} \int_{E_{\min}}^{E_{\max}} f(E) P^m_{\nu_e \to \nu_\mu}(E) \frac{dE}{E_\mu} \tag{6a}$$

$$n(\bar{\nu}_\mu) = N_{\mu^-} N_{\mathrm{kT}} \frac{10^9 N_A}{m_\mu^2 \pi} \frac{E_\mu^3}{L^2} \int_{E_{\min}}^{E_{\max}} \bar{f}(E) P^m_{\bar{\nu}_e \to \bar{\nu}_\mu}(E) \frac{dE}{E_\mu}, \tag{6b}$$

where

$$f(E) = g_{\nu_e}(E/E_\mu)(\sigma_{\nu_\mu}(E)/E_\mu)\epsilon_{\mu^-}(E), \tag{7a}$$

$$\bar{f}(E) = g_{\bar{\nu}_e}(E/E_\mu)(\sigma_{\bar{\nu}_\mu}(E)/E_\mu)\epsilon_{\mu^+}(E) \tag{7b}$$

are weight functions taking into account the energy spectrum (normalized to 1) of the electron neutrinos (antineutrinos) produced in the $\mu^+$ ($\mu^-$) decay, $g_{\nu_e(\bar{\nu}_e)}(E/E_\mu)$, the charged current cross section per nucleon, $\sigma_{\nu_\mu(\bar{\nu}_\mu)}(E)$, and the efficiency for the detection of $\mu^-$ ($\mu^+$), $\epsilon_{\mu^-(\mu^+)}(E)$[2]. $P^m_{\nu_e \to \nu_\mu}(E)$ ($P^m_{\bar{\nu}_e \to \bar{\nu}_\mu}(E)$) is the oscillation probability for neutrinos travelling inside the earth taking into account matter effects. Finally $N_{\mathrm{kT}}$ is the size of the detector in kilotons and $N_{\mu^\pm}$ is the number of "useful" $\mu^\pm$ decays, namely the number of decays occurring in the straight section of the storage ring pointing to the detector. In the numerical calculations we use

$$g_{\nu_e}(x) = g_{\bar{\nu}_e}(x) = 12x^2(1-x), \tag{8a}$$

$$\sigma_{\nu_\mu}(E) = 0.67 \cdot 10^{-38} E\, \mathrm{cm}^2/\mathrm{GeV}, \qquad \sigma_{\bar{\nu}_\mu}(E) = 0.34 \cdot 10^{-38} E\, \mathrm{cm}^2/\mathrm{GeV}, \tag{8b}$$

$$\text{and} \quad \epsilon_{\mu^\pm}(E) = 30\% \quad \text{for } E > 5\,\mathrm{GeV}, \tag{8c}$$

so that $f(E)/\bar{f}(E) = 2$ (independently of the energy). We also apply a lower cut $E_{\min} = 5\,\mathrm{GeV}$ on the neutrino energies in order to have a good detection efficiency in all the energy range[3].

# 3 Measuring CP-violation

## 3.1 Experimental apparatus and matter asymmetries

The asymmetry between the number of $\nu_\mu$ and $\bar{\nu}_\mu$ events seen in the detector, normalized to the initial number of muon decays is given by

$$\hat{a}_{\mathrm{tot}} = \frac{n(\nu_\mu)/N_{\mu^+} - n(\bar{\nu}_\mu)/N_{\mu^-}}{n(\nu_\mu)/N_{\mu^+} + n(\bar{\nu}_\mu)/N_{\mu^-}}. \tag{9}$$

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[2] We neglect here the finite resolution of the detector.

[3] The numerical results we obtain agree with those obtained in [5] if the efficiency is set to 100% and no lower cut is used.

An "intrinsic" leptonic CP-violation associated with a non-vanishing phase $\delta$ in (2) would contribute to give a non-vanishing $\hat{a}_{\rm tot}$. However, even if there were not any intrinsic CP-violation, $\hat{a}_{\rm tot}$ would still be non-vanishing due to the CP-asymmetry of the experimental apparatus, $f \neq \bar{f}$ in (6), and due to matter effects, that affect neutrinos and antineutrinos in different ways. In other words, $\hat{a}_{\rm tot}$ is an estimator of

$$a_{\rm tot} = \frac{\int P^m_{\nu_e\to\nu_\mu}(E) f(E)\, dE - \int P^m_{\bar{\nu}_e\to\bar{\nu}_\mu}(E) \bar{f}(E)\, dE}{\int P^m_{\nu_e\to\nu_\mu}(E) f(E)\, dE + \int P^m_{\bar{\nu}_e\to\bar{\nu}_\mu}(E) \bar{f}(E)\, dE}, \tag{10}$$

that can be non-vanishing even when $\delta = 0$ due to $f \neq \bar{f}$ and $P^m_{\nu_e\to\nu_\mu}(E,L) \neq P^m_{\bar{\nu}_e\to\bar{\nu}_\mu}(E,L)$.

Getting rid of the experimental apparatus asymmetry is easy. It is sufficient to weight e.g. each $\bar{\nu}$ event with $f(E)/\bar{f}(E)$, where $E$ is the energy of the event, namely to replace $n(\bar{\nu}_\mu)$ in (9) with

$$n'(\bar{\nu}_\mu) \equiv \sum_{i=1}^{n(\bar{\nu}_\mu)} \frac{f(E_i)}{\bar{f}(E_i)}, \tag{11}$$

where $E_i$ is the energy of the $i$-th $\bar{\nu}_\mu$ event. In other words, instead of $\hat{a}_{\rm tot}$ we consider

$$\hat{a}_{\rm CP+m} = \frac{n(\nu_\mu)/N_{\mu^+} - n'(\bar{\nu}_\mu)/N_{\mu^-}}{n(\nu_\mu)/N_{\mu^+} + n'(\bar{\nu}_\mu)/N_{\mu^-}}, \tag{12}$$

that estimates a quantity independent of the experimental CP-violation,

$$a_{\rm CP+m} = \frac{\int (P^m_{\nu_e\to\nu_\mu}(E) - P^m_{\bar{\nu}_e\to\bar{\nu}_\mu}(E)) f(E)\, dE}{\int (P^m_{\nu_e\to\nu_\mu}(E) + P^m_{\bar{\nu}_e\to\bar{\nu}_\mu}(E)) f(E)\, dE}. \tag{13}$$

We remark, however, that the subtraction of the experimental asymmetry introduces an unavoidable systematic error due to the uncertainties in the knowledge of the flux, cross section and efficiency ratios in $f(E)/\bar{f}(E)$.

In our numerical calculations $n'(\bar{\nu}_\mu)$ is simply given by

$$n'(\bar{\nu}_\mu) = 2n(\bar{\nu}_\mu), \tag{14}$$

since $f(E)/\bar{f}(E) = 2$ independently of the neutrino energy $E$. In this case, using (12) is equivalent to using a different normalization for $n(\nu_\mu)$ and $n(\bar{\nu}_\mu)$ in (9). However, if $f(E)/\bar{f}(E)$ had a small dependence on the energy due e.g. to $\epsilon_{\mu^-}(E)/\epsilon_{\mu^+}(E)$, using (12) would allow a more accurate subtraction of the experimental asymmetry.

The quantity $\hat{a}_{\rm CP+m}$ in (12) still contains a pollution due to the matter asymmetry. In principle the best attitude towards such a pollution would be trying to experimentally disentangle matter from genuine CP-violation effects. A detailed study of the feasibility of this possibility is beyond the aim of this paper. Here we would rather cope with matter effects by calculating them [1]. An experimental confirmation of the reliability of such a theoretical calculation, besides being important in itself, would be welcome in order to support this method. Other theoretical approaches to the issue of matter effects can be found in [13].

First of all, we define the matter asymmetry in absence of CP-violation as

$$a_{\mathrm{m}} = \frac{\int (P^{m,\delta=\mathbf{0}}_{\nu_e\to\nu_\mu}(E) - P^{m,\delta=\mathbf{0}}_{\bar\nu_e\to\bar\nu_\mu}(E)) f(E)\, dE}{\int (P^{m,\delta=\mathbf{0}}_{\nu_e\to\nu_\mu}(E) + P^{m,\delta=\mathbf{0}}_{\bar\nu_e\to\bar\nu_\mu}(E)) f(E)\, dE}, \tag{15}$$

where $P^{m,\delta=\mathbf{0}}_{\nu_e\to\nu_\mu}$, $P^{m,\delta=\mathbf{0}}_{\bar\nu_e\to\bar\nu_\mu}$ are the oscillation probabilities calculated for $\delta = 0$ but taking into account matter effects. Then, we define the CP-asymmetry in absence of matter effects as

$$a_{\mathrm{CP}} = \frac{\int (P_{\nu_e\to\nu_\mu}(E) - P_{\bar\nu_e\to\bar\nu_\mu}(E)) f(E)\, dE}{\int (P_{\nu_e\to\nu_\mu}(E) + P_{\bar\nu_e\to\bar\nu_\mu}(E)) f(E)\, dE} = \frac{\int P_{\not{CP}}(E) f(E)\, dE}{\int P_{CP}(E) f(E)\, dE}, \tag{16}$$

where $P_{\nu_e\to\nu_\mu}$, $P_{\bar\nu_e\to\bar\nu_\mu}$ are the oscillation probabilities computed in vacuum but taking into account CP-violation effects. Now, despite $a_{\mathrm{CP+m}}$ is not simply given by the sum of $a_{\mathrm{CP}}$ and $a_{\mathrm{m}}$, it turns out that for practical purposes the relation

$$a_{\mathrm{CP+m}} \simeq a_{\mathrm{CP}} + a_{\mathrm{m}} \tag{17}$$

is a very good approximation if $a_{\mathrm{m}}$ is not too large[4]. In any case, the error one makes recovering $a_{\mathrm{CP}}$ through (17), $a_{\mathrm{CP}} = a_{\mathrm{CP+m}} - a_{\mathrm{m}}$, is smaller than the uncertainties on $a_{\mathrm{m}}$ itself. The latter therefore constitute the main source of systematic error from matter effects. In conclusion, as a measure of CP-violation we use the quantity

$$\hat{a}_{\mathrm{CP}} = \frac{n(\nu_\mu)/N_{\mu^+} - n'(\bar\nu_\mu)/N_{\mu^-}}{n(\nu_\mu)/N_{\mu^+} + n'(\bar\nu_\mu)/N_{\mu^-}} - a_{\mathrm{m}}. \tag{18}$$

Being $\hat{a}_{\mathrm{CP}}$ an estimator of $a_{\mathrm{CP}}$ in (16), measuring a non-vanishing value of $\hat{a}_{\mathrm{CP}}$ would indicate a genuine leptonic CP-violation.

### 3.2 Systematic and statistical errors

In this Section we discuss the systematic and statistical errors involved in a measurement of $a_{\mathrm{CP}}$ through $\hat{a}_{\mathrm{CP}}$. There are different sources of systematic errors in $\hat{a}_{\mathrm{CP}}$. One is given by the uncertainties on the experimental asymmetries, $g_{\nu_e}/g_{\bar\nu_e}$, $\sigma_{\nu_\mu}/\sigma_{\bar\nu_\mu}$ and $\epsilon_{\mu^-}/\epsilon_{\mu^+}$, that enter $\hat{a}_{\mathrm{CP}}$ through $f/\bar{f}$ in (11). Another one is given by the uncertainties on the matter asymmetry, in turn due to the uncertainties on the neutrino mass and mixing parameters we use to calculate it. Here we concentrate on the latter, assuming that the former will be made small enough by experimental determinations of $f/\bar{f}$.

Whatever is the strategy used towards the matter asymmetry, it is clear that the smaller the matter effects are the cleaner a possible measurement of a genuine violation of CP would be. Since the matter asymmetry grows with the baseline faster than the CP-asymmetry, $a_{\mathrm{m}}$ constitutes the main limitation to the length of the baseline of an experiment aiming at measuring a violation of CP. In order to illustrate and make quantitative the latter point we can use the approximation one gets for $P^m_{\nu_e\to\nu_\mu}$, $P^m_{\bar\nu_e\to\bar\nu_\mu}$ in the

[4] This is essentially because the corrections to (17) arise only at second order in the asymmetries and is confirmed by a numerical analysis [1].

limit $\Delta m^2_{21} = 0$:

$$P^m_{\nu_e \to \nu_\mu} = \sin^2\theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(\Delta_{32} c_+^{1/2})}{c_+} \tag{19a}$$

$$P^m_{\bar\nu_e \to \bar\nu_\mu} = \sin^2\theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(\Delta_{32} c_-^{1/2})}{c_-}, \tag{19b}$$

where

$$c_\pm = (1 \mp x)^2 \pm 4x \sin^2\theta_{13} \simeq (1 \mp x)^2, \quad x = \frac{2EV}{\Delta m^2_{32}} \tag{20}$$

and $V$ is the matter induced potential. We then get

$$a_{\rm m} = \frac{\sin^2(\Delta_{32} c_+^{1/2})/c_+ - \sin^2(\Delta_{32} c_-^{1/2})/c_-}{\sin^2(\Delta_{32} c_+^{1/2})/c_+ + \sin^2(\Delta_{32} c_-^{1/2})/c_-} \simeq \frac{V \Delta m^2_{32}}{12} \frac{L^2}{E} (1 - 2\sin^2\theta_{13}) \tag{21}$$

for the matter asymmetry before integration on the energy range, or

$$a_{\rm m} \simeq 0.7 \cdot 10^{-6} \frac{L^2(\mathrm{km})}{E(\,\mathrm{GeV})} \frac{\Delta m^2_{32}}{3 \cdot 10^{-3}\,\mathrm{eV}^2}. \tag{22}$$

The expansion in (21) holds when $L \lesssim \pi/V$ and $\Delta_{32} \lesssim \pi/2$. When these conditions are not fulfilled the matter asymmetry is large or quickly oscillating and therefore out of control. However, we stress that the main corrections to (22) come from the approximation (19), particularly rough when $\theta_{13}$ is very small, rather than from the expansion in (21). Nonetheless, Eq. (22) is sufficient for our illustrative purposes (exact formulae are used in all numerical calculations). When compared with the behavior of $a_{\rm CP}$ with $L$ and $E$,

$$a_{\rm CP} \propto \sin\Delta_{21} \propto \frac{L}{E}, \tag{23}$$

Eq. (22) shows that $|a_{\rm m}/a_{\rm CP}|$ grows with the baseline length $L$ (as far as $L \lesssim \pi/V$ and $\Delta_{32} \lesssim \pi/2$). Very long baselines are therefore more suitable for the study of matter effects than for CP-violation. We will come back to this point in Section 4.

Before describing into detail how we work out the systematic error due to matter effects, we remark that Eq. (22), besides illustrating the linear dependence of $a_{\rm m}$ on $\Delta m^2_{32}$, makes also explicit its dependence on the sign of $\Delta m^2_{32}$. The knowledge of that sign is therefore essential in order to properly subtract matter effects and will be assumed here. A detailed analysis of how that sign could be determined through a measurement of matter effects is beyond the scope of this paper.

We estimate the systematic error $\delta_{\rm syst} a_{\rm CP}$ on $a_{\rm CP}$ due to the uncertainty on $a_{\rm m}$ by scanning the ranges of the parameters on which $a_{\rm m}$ depends. $\delta_{\rm syst} a_{\rm CP}$ is then obtained as the half-width of the range of values assumed by $a_{\rm m}$. As for the $\Delta m^2_{21}$, $\theta_{12}$, $\theta_{23}$ ranges, less important in the determination of $\delta_{\rm syst} a_{\rm CP}$, we assume, to be conservative, that they will not differ very much from the present ones [8, 9, 10, 11]:

$$\Delta m^2_{21} = (0.2 - 3.0) \cdot 10^{-4}\,\mathrm{eV}^2, \quad \sin^2 2\theta_{12} = (0.6 - 0.96), \quad \sin^2 2\theta_{23} > 0.8 \tag{24}$$

(all at 99% CL; the $\Delta m^2_{21}$, $\theta_{12}$ values correspond of course to the large mixing angle solution). It is less clear which ranges should be used for the more crucial $\Delta m^2_{32}$ and $\theta_{13}$. The next generation of long-baseline experiments will be able to provide information on the latter parameters and will reduce the uncertainties on them. However, it has been shown in [1] that, for favourable values of $\Delta m^2_{21}$ in the large angle range in (24) and for values of $\Delta m^2_{32}$ and $\theta_{13}$ within the sensitivity of an experiment like e.g. MINOS, the CP-violating part of the oscillation probability could constitute up to 30–40% of the total oscillation probability. This in turn means that, unless some compelling evidence excluding the large angle solution is found, a systematic uncertainty up to 30–40% could affect a $\sin^2 2\theta_{13}$ determination made measuring a single oscillation probability. A precise determination of $\theta_{13}$ would on the contrary require an experiment able to measure both the $\nu_e \leftrightarrow \nu_\mu$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillation probabilities, namely able to measure CP-violation. A neutrino factory would provide this possibility. Once both $P_{\nu_e\to\nu_\mu}$ and $P_{\bar{\nu}_e\to\bar{\nu}_\mu}$ were measured, the CP-conserving part of the probability could be recovered as $P_{CP} = (P_{\nu_e\to\nu_\mu} + P_{\bar{\nu}_e\to\bar{\nu}_\mu})/2$. This would not completely remove the ambiguities associated to the phase $\delta$, since $P_{CP}$ depends on $\delta$ as well. An estimate of $\delta$ through $a_{\rm CP}$ would therefore be also welcome in order to get rid of such ambiguities. Here, for the purpose of estimating $\delta_{\rm syst} a_{\rm CP}$ we will assume that a determination of $\Delta m^2_{32}$ and $\sin^2 2\theta_{13}$ with a precision higher than 20% will become available.

The discussion of statistical uncertainties is straightforward. In order to exclude the possibility that a measurement of a non-vanishing $\hat{a}_{\rm CP}$ is due to a statistical fluctuation, the measured value must be larger than $n_\sigma \delta_{\rm stat} a_{\rm CP}$, where $\delta_{\rm stat} a_{\rm CP}$ is the "$1\sigma$" statistical error on $a_{\rm CP}$ in absence of CP-violation and $n_\sigma$ is the number of standard deviations we require. Since in absence of CP-violation the expectations of $n(\nu_\mu)/N_{\mu^+}$ and $n'(\bar{\nu}_\mu)/N_{\mu^-}$ are equal, we get

$$\delta_{\rm stat} a_{\rm CP} = \left( \frac{1}{4\left\langle n(\nu_\mu)\right\rangle} + \frac{1}{4\left\langle n(\bar{\nu}_\mu)\right\rangle} \right)^{1/2}, \tag{25}$$

where $\langle n(\nu_\mu)\rangle$ and $\langle n(\bar{\nu}_\mu)\rangle$ are the expected number of $\nu_\mu$ and $\bar{\nu}_\mu$ interactions seen in the detector. When deciding whether it would be possible to measure a violation of CP in a given point of parameter space, we will assume as usual that the measured value of $\hat{a}_{\rm CP}$ coincides with the value of $a_{\rm CP}$ expected in that point.

Finally, let us shortly discuss the contribution of the background to the statistical error. According to the present estimates, the main source of background is due to charm production in the charged and neutral current neutrino interactions in the detector [14]. For example, the charged current background is due to:

$$\mu^- \to \nu_\mu \xrightarrow{\text{no osc.}} \nu_\mu \xrightarrow{\text{CC int.}} (\mu^-)_{\rm lost} + c \xrightarrow{c\text{-decay}} (\mu^+)_{\rm found}. \tag{26}$$

This background only affects the signal corresponding to the $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillation:

$$\mu^- \to \bar{\nu}_e \xrightarrow{\text{osc.}} \bar{\nu}_\mu \xrightarrow{\text{CC int.}} \mu^+. \tag{27}$$

The background subtraction corresponds to a replacement

$$n(\bar{\nu}_\mu) \to n(\bar{\nu}_\mu) - (n(\bar{\nu}_\mu))_{\rm back} \tag{28}$$

in Eq. (18). Such a subtraction introduces a further source of statistical error. Using the estimate [14]

$$(n(\bar{\nu}_\mu))_{\text{back}} \sim 10^{-5} n'(\bar{\nu}_\mu)_{P=1}, \tag{29}$$

where $n(\bar{\nu}_\mu)_{P=1}$ stands for the number of $\bar{\nu}_\mu$ interactions that would be seen if all the initial $\bar{\nu}_e$ oscillated into $\bar{\nu}_\mu$, we get the complete expression for the statistical error we will use:

$$\delta_{\text{stat}} a_{\text{CP}} = \left( \frac{1}{4\langle n(\nu_\mu)\rangle} + \frac{1}{4\langle n(\bar{\nu}_\mu)\rangle} + \frac{10^{-5}\langle n(\bar{\nu}_\mu)_{P=1}\rangle}{4\langle n(\bar{\nu}_\mu)\rangle^2} \right)^{1/2}. \tag{30}$$

Note, however, that in a long-baseline experiment with $\sin^2\Delta_{32} = \mathcal{O}(1)$ the background contribution in (30) is not negligible only for $\sin^2 2\theta_{13} \lesssim 10^{-5}$.

## 4 Capability of the neutrino factory

In this Section we show in which part of the parameter space an evidence of leptonic CP-violation could be obtained as a function of the relevant experimental parameters. In order to obtain an evidence for a non-vanishing

$$a_{\text{CP}} = \frac{\int (P_{\nu_e\to\nu_\mu}(E) - P_{\bar{\nu}_e\to\bar{\nu}_\mu}(E)) f(E)\, dE}{\int (P_{\nu_e\to\nu_\mu}(E) + P_{\bar{\nu}_e\to\bar{\nu}_\mu}(E)) f(E)\, dE} = \frac{\int P_{\cancel{CP}}(E) f(E)\, dE}{\int P_{CP}(E) f(E)\, dE}, \tag{16}$$

a value of $\hat{a}_{\text{CP}}$ larger than the systematic and statistical errors on it should be measured:

$$|\hat{a}_{\text{CP}}| > \delta_{\text{syst}} a_{\text{CP}}, \qquad |\hat{a}_{\text{CP}}| > n_\sigma \delta_{\text{stat}} a_{\text{CP}}. \tag{31}$$

In the previous equation, $\delta_{\text{syst}} a_{\text{CP}}$ is calculated as described in Section 3.2, $\delta_{\text{stat}} a_{\text{CP}}$ is the $1\sigma$ statistical error given by (30) and $n_\sigma$ is the number of standard deviations corresponding to the CL we want to achieve (we will use $n_\sigma = 5$ as a reference value).

The experimental parameters we consider are

- the size of the detector in kT, $N_{\text{kT}}$;
- the total number of useful muon decays $N_\mu = N_{\mu^+} + N_{\mu^-}$;
- the ratio of $\mu^+$ versus $\mu^-$ decays $N_{\mu^+}/N_{\mu^-}$ for a given total number of muon decays $N_\mu = N_{\mu^+} + N_{\mu^-}$;
- the energy of the stored muons $E_\mu$ and the lower and upper cuts on the neutrino energies, $E_{\text{min}}$ and $E_{\text{max}}$;
- the baseline $L$.

First, we briefly identify a convenient choice of $N_{\mu^+}/N_{\mu^-}$ and $E_\mu$, $E_{\text{min}}$, $E_{\text{max}}$. Then, using that choice, we show which is the portion of parameter space where the asymmetry could be measurable as a function of the product $N_\mu N_{\text{kT}}$ and for two choices of the baseline, $L = 732\,\text{km}$ and $L = 3000\,\text{km}$.

The dependence of the statistical error on $N_{\mu^+}$ and $N_{\mu^-}$ is simple. Neglecting the small background contribution and using Eqs. (8) we find

$$\delta_{\rm stat} a_{\rm CP} \propto \left( \frac{1}{N_{\mu^+}} + \frac{1}{N_{\mu^-}} \frac{\sigma_{\nu_\mu}}{\sigma_{\bar{\nu}_\mu}} \right)^{1/2} , \tag{32}$$

where $\sigma_{\nu_\mu}/\sigma_{\bar{\nu}_\mu} \simeq 2$. When $N_{\mu^+} = N_{\mu^-}$ the $\bar{\nu}_\mu$ events are less than the $\nu_\mu$ events because of the smaller $\bar{\nu}_\mu$ cross section. Therefore, their contribution to the statistical error is larger. For a given total number of available muon decays, $N_\mu = N_{\mu^+} + N_{\mu^-}$, the statistical error can be made smaller by choosing a ratio $N_{\mu^+}/N_{\mu^-}$ smaller than one. Here we therefore use the $N_{\mu^+}/N_{\mu^-}$ ratio obtained by minimizing (32), namely

$$N_{\mu^+} = \frac{\sqrt{\sigma_{\bar{\nu}_\mu}}}{\sqrt{\sigma_{\nu_\mu}} + \sqrt{\sigma_{\bar{\nu}_\mu}}} N_\mu \simeq 0.4\, N_\mu \tag{33a}$$

$$N_{\mu^-} = \frac{\sqrt{\sigma_{\nu_\mu}}}{\sqrt{\sigma_{\nu_\mu}} + \sqrt{\sigma_{\bar{\nu}_\mu}}} N_\mu \simeq 0.6\, N_\mu . \tag{33b}$$

Unfortunately, using (33) instead of $N_{\mu^+} = N_{\mu^-}$ only improves $\delta_{\rm stat} a_{\rm CP}$ by a few percents.

Let us now discuss the energy parameters. First of all, we use $E_{\rm min} = 5\,\text{GeV}$ as a lower cut on the neutrino energies in order to have an efficient muon detection in all the energy range. An upper cut is also necessary since $a_{\rm CP}/\delta_{\rm stat} a_{\rm CP} \propto 1/\sqrt{E_{\rm max}}$ when $E_{\rm max}$ is sufficiently high [5]. The upper cut will of course be lower than the muon energy in the storage ring. The low energy neutrino rates do not strongly depend on the value of $E_\mu \geq E_{\rm max}$, as far as $E_\mu$ is in the range presently discussed $10\,\text{GeV} < E_\mu < 50\,\text{GeV}$. However, this dependence is not negligible either. For example, the rate of charged current interactions of low energy neutrinos in the detector is inversely proportional to $E_\mu$, as it can easily seen by using Eq. (8).

The optimization of $E_{\rm max}$ and $E_\mu$ is shown in Fig. 1 for the two baselines we consider, $L = 732\,\text{km}$ (Fig. 1a) and $L = 3000\,\text{km}$ (Fig. 1b). The values of $E_{\rm max}$ and $E_\mu$ that maximize $|a_{\rm CP}/\delta_{\rm stat} a_{\rm CP}|$ are shown by the black spots in the $E_\mu$–$E_\mu/E_{\rm min}$ planes of Fig. 1. The three contour lines around the spots surround the regions where $|a_{\rm CP}/\delta_{\rm stat} a_{\rm CP}|$ is larger than 80%, 60%, 40% of the optimal value respectively. The figures show that there is a certain freedom in choosing the values of $E_{\rm max}$ and $E_\mu$. In what follows we will use the optimal values $E_{\rm max} = E_\mu = 20\,\text{GeV}$ in the $L = 732\,\text{km}$ case and $E_{\rm max} = E_\mu = 40\,\text{GeV}$ in the $L = 3000\,\text{km}$ one. However, using e.g. $E_{\rm max} = 20\,\text{GeV}$ and $E_\mu = 50\,\text{GeV}$ for $L = 732\,\text{km}$ would involve a reduction of $|a_{\rm CP}/\delta_{\rm stat} a_{\rm CP}|$ of 20% only. The dependence of these results on the neutrino parameters is negligible.

Having described the values of $N_{\mu^+}/N_{\mu^-}$ $E_\mu$, $E_{\rm min}$, $E_{\rm max}$ we will use, we now move to the discussion of what amount of parameter space could be covered by using a total number of $N_\mu = N_{\mu^+} + N_{\mu^-}$ useful muon decays and a $N_{\rm kT}$ kT detector. First of all we describe the parameter space. The neutrino parameters are $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\Delta m^2_{21}$, $\Delta m^2_{32}$, $\delta$. The constraints on them we use are those obtained by the standard fits of the atmospheric and solar neutrino data [8, 9, 10, 11] taking into account the constraints given by the CHOOZ experiment [12]. Since the uncertanties on $\theta_{12}$ and $\theta_{23}$ do not significantly affect the results, we set $\sin^2 2\theta_{12} = 0.8$ [9] and $\sin^2 2\theta_{23} = 1$. The dependence of the results on $\delta$ is trivial since $a_{\rm CP}$ is to a good approximation linear in $\sin\delta$. This is because $P_{\not{CP}}$ is

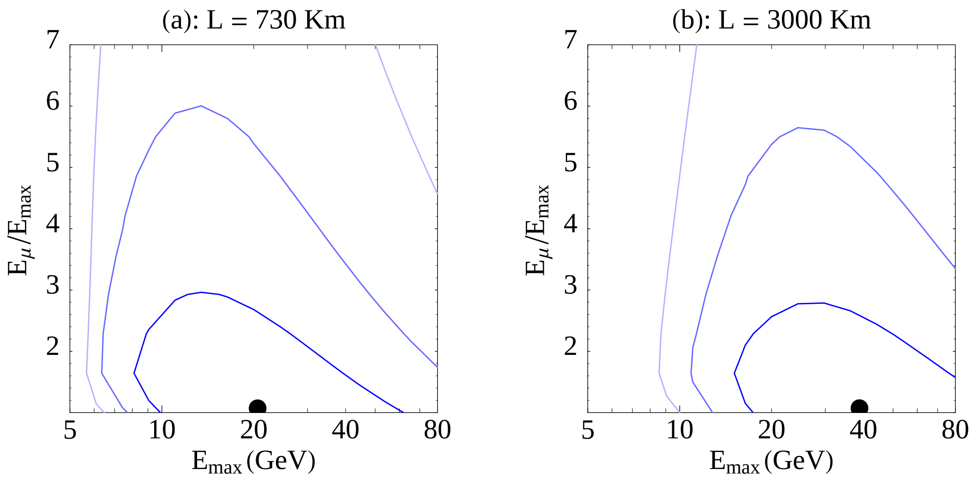


**Figure 1**: *Optimization of $E_{max}$ and $E_\mu/E_{max}$ in the case of a long baseline of $L = 732\,km$, (a), and of a very long baseline of $L = 3000\,km$, (b). The black spots indicate the values of $E_{max}$ and $E_\mu/E_{max}$ that give the best relative statistical error. The three contour lines surround the regions where the relative statystical error is 80%, 60%, 40% of the optimal value respectively.*

strictly linear in $\sin\delta$ and $P_{CP}$ depends on $\delta$ only through sub-leading $\Delta m^2_{21}$ terms[5]. We are therefore led to consider a three-dimensional parameter space described by

$$10^{-3}\,\mathrm{eV}^2 \leq |\Delta m^2_{32}| \leq 10^{-2}\,\mathrm{eV}^2 \ [8,\ 9] \tag{34a}$$

$$2\cdot 10^{-5}\,\mathrm{eV}^2 \leq \Delta m^2_{21} \leq 3\cdot 10^{-4}\,\mathrm{eV}^2 \ [10] \tag{34b}$$

$$\theta_{13} \leq \text{CHOOZ and 3-}\nu \text{ fit limits [11].} \tag{34c}$$

Since the CP-violating effects crucially depend on $\Delta m^2_{21}$, we plot our results in the $\sin^2 2\theta_{13}$–$\Delta m^2_{21}$ plane for different values of $|\Delta m^2_{32}|$. We first show and discuss the results for the $L = 732\,\mathrm{km}$ baseline.

### 4.1 The $L = 732\,$km baseline

In Fig. 2 we show the portion of the $\sin^2 2\theta_{13}$–$\Delta m^2_{21}$ parameter space where an evidence of CP-violation could be obtained for three different values of $|\Delta m^2_{32}|$: the present central value, $|\Delta m^2_{32}| = 3\cdot 10^{-3}\,\mathrm{eV}^2$ (Fig. 2b in the center), and two values at the borders of the presently allowed region, $|\Delta m^2_{32}| = 10^{-3}\,\mathrm{eV}^2$ (Fig. 2a) and $|\Delta m^2_{32}| = 10^{-2}\,\mathrm{eV}^2$ (Fig. 2c). The rectangular windows inside the light shadowed areas represent the part of parameter space allowed by the constraints (34). The dark shadowed parts of the plots represent the portion of parameter space where an evidence of CP-violation could not be obtained because the systematic error exceeds the asymmetry itself even for maximal CP-violation. We see that the systematic error due to matter effects does not represent a serious problem for the $L = 732\,\mathrm{km}$ baseline, especially when $|\Delta m^2_{32}|$ is low. Note that the size of this

[5] Moreover, the CP-conserving quantity $P_{CP}$ depends on $\delta$ only through $\cos\delta$, so that the dependence on $\delta$ is particularly mild when the CP-violation is large.

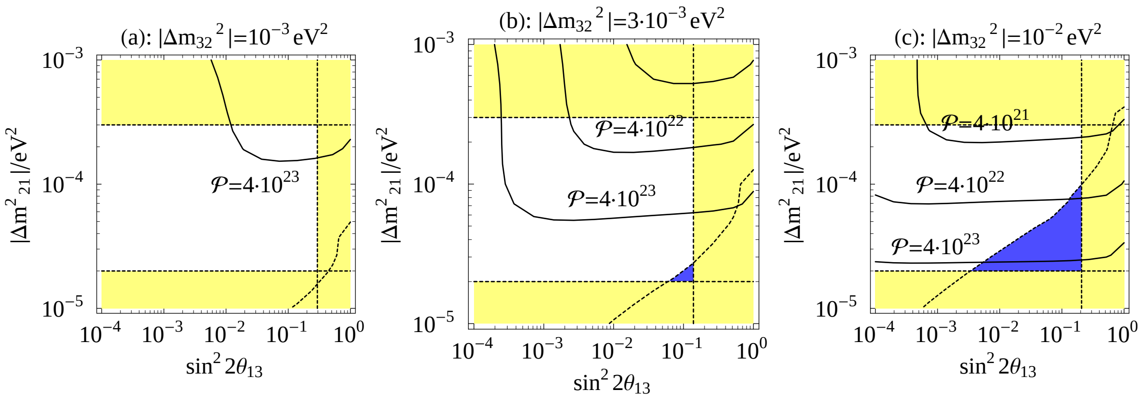


**Figure 2**: *Capability of a neutrino factory using a baseline of $L = 732$ km in the $\sin^2 2\theta_{13}$–$\Delta m^2_{21}$ plane for $|\Delta m^2_{32}| = 10^{-3}\,\mathrm{eV}^2$, (a), $|\Delta m^2_{32}| = 3 \cdot 10^{-3}\,\mathrm{eV}^2$, (b), and $|\Delta m^2_{32}| = 10^{-2}\,\mathrm{eV}^2$, (c). The rectangular windows inside the light-shadowed areas correspond to the constraints (34). In the dark-shadowed regions the systematic error on $a_{CP}$ due to matter effects exceeds $|a_{CP}|$. The size of these regions depends on the precision of the future determination of $|\Delta m^2_{32}|$ and $\theta_{13}$. In order to cover the portion of parameter space above the solid line corresponding to a given value of $\mathcal{P}$, values of $N_\mu$, $N_{kT}$ and $\epsilon_{\mu^\pm}$ such that $N_\mu N_{kT}/(n_\sigma/5)^2(\epsilon_{\mu^\pm}/30\%)\sin^2\delta > \mathcal{P}$ should be used.*

systematic error depends on the precision of the future determinations of $|\Delta m^2_{32}|$ and $\theta_{13}$. If precisions higher than those assumed here could be achieved, the dark-shadowed regions in Fig. 2 would be smaller. Fig. 2 shows that the relative systematic error $|\delta_{\mathrm{syst}} a_{\mathrm{CP}}/a_{\mathrm{CP}}|$ grows with $|\Delta m^2_{32}|$. This is because $a_{\mathrm{CP}}$ is almost independent of $\Delta m^2_{32}$ when $\theta_{13}$ is not too small [1], whereas $|\delta_{\mathrm{syst}} a_{\mathrm{CP}}|$, roughly proportional to $|a_{\mathrm{m}}|$, grows with $|\Delta m^2_{32}|$, as illustrated by Eq. (22).

The region of the parameter space where the statistics is sufficient to exclude a fluctuation with a CL corresponding to $n_\sigma$ standard deviations is shown in the plots of Fig. 2 for different values of the number of useful muon decays $N_\mu$, of the size of the detector in kT, $N_{\mathrm{kT}}$, and of the size of CP-violation, $|\sin\delta|$ (regions above the solid lines). Since $a_{\mathrm{CP}} \simeq a_{\mathrm{CP}}(\delta = \pi/2)\cdot\sin\delta$ and since $\delta_{\mathrm{stat}} a_{\mathrm{CP}} \propto 1/\sqrt{N_\mu N_{\mathrm{kT}}}$, the regions shown actually depend on the combination

$$\mathcal{P} = \frac{N_\mu N_{\mathrm{kT}}}{(n_\sigma/5)^2}\cdot\frac{\epsilon_{\mu^\pm}}{30\%}\cdot\sin^2\delta \tag{35}$$

only. For a 30% efficiency, maximal CP-violation and a CL corresponding to the reference value $n_\sigma = 5$, $\mathcal{P}$ is simply the product of $N_\mu$ and $N_{\mathrm{kT}}$. The three solid lines in Figs. 2 (one in Fig. 2a) correspond to three possible values of $\mathcal{P}$: the smallest one, $\mathcal{P} = 4\cdot 10^{21}$, corresponds (for $\sin^2\delta = 1$ and $n_\sigma = 5$) to the reference set-up considered in [3], a 10 kT detector and $2\cdot 10^{20}$ useful muon decays in both the CP-conjugated channels. The intermediate value, $\mathcal{P} = 4\cdot 10^{22}$, corresponds e.g. to an improvement of one order of magnitude in the intensity of the muon source. The highest value, $\mathcal{P} = 4\cdot 10^{23}$, requires a further improvement, e.g. a 30 kT detector with very high efficiency. Some of these options are currently under discussion [15].

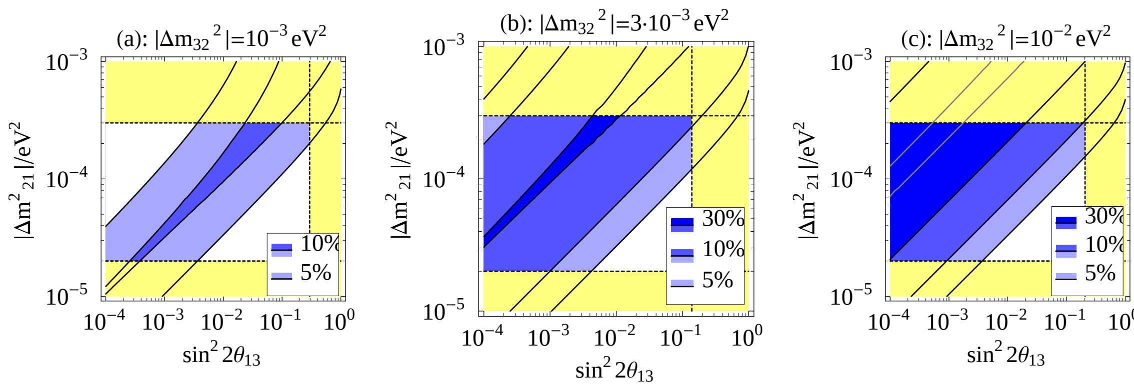


**Figure 3**: *Contour lines of* $|a_{CP}(\delta = \pi/2)| \simeq |a_{CP}/\sin\delta|$ *in the* $\sin^2 2\theta_{13}$–$\Delta m^2_{21}$ *plane for* $|\Delta m^2_{32}| = 10^{-3}\,\mathrm{eV}^2$, *(a),* $|\Delta m^2_{32}| = 3 \cdot 10^{-3}\,\mathrm{eV}^2$, *(b), and* $|\Delta m^2_{32}| = 10^{-2}\,\mathrm{eV}^2$, *(c), in the case of a* $L = 732\,km$ *baseline. In (c) the contour lines inside the darker region correspond to a 70% asymmetry.*

Fig. 2 clearly shows that the sensitivity lines strongly depend on the value of $|\Delta m^2_{32}|$. This is because the statistics, proportional to $\sin^2\Delta_{32}$, grows with $|\Delta m^2_{32}|$, whereas the asymmetry is almost independent of $\Delta m^2_{32}$ for $\theta_{13}$ not too small [1]. A $|\Delta m^2_{32}|$ value around the present central value would allow to cover a major part of the parameter space with the highest value of $\mathcal{P}$. If $|\Delta m^2_{32}|$ were at the top of the presently allowed range, that value would allow to cover essentially all the parameter space, while the intermediate value of $\mathcal{P}$ in this case would be enough to cover a good half of it. At the bottom of the $|\Delta m^2_{32}|$ range getting an evidence of leptonic CP-violation would only be possible in a small part of the parameter space even with the highest neutrino fluxes, unless a longer baseline is used. This possibility will be considered in the next Subsection.

Values of $\Delta m^2_{21}$ above the range (34b) are allowed in "non-standard" analysis of solar neutrino data where the possibility of an unknown systematic uncertainty in one of the solar experiments is taken into consideration [16]. Fig. 2 shows that relatively low values of $\mathcal{P}$ could cover the corresponding region.

For completeness, we show in Fig. 3 three contour plots of $|a_{\mathrm{CP}}(\delta = \pi/2)| \simeq |a_{\mathrm{CP}}/\sin\delta|$ in the same sections of the three-dimensional parameter space as in Fig. 2. The asymmetry values corresponding to the contour lines are specified in the legends. The structure of the plots is easily explained: as far as the $\Delta m^2_{21}$ effects in the CP-conserving part of the oscillation probability can be neglected,

$$P_{\not{CP}} \propto \sin 2\theta_{13} \qquad \text{and} \qquad P_{CP} \propto \sin^2 2\theta_{13}, \tag{36}$$

so that the CP-asymmetry gets larger when $\theta_{13}$ gets smaller, $a_{\mathrm{CP}} \propto 1/\sin 2\theta_{13}$ [1]. However, at some point the smallness of $\theta_{13}$ makes the $\Delta m^2_{21}$ effects in $P_{CP}$ important so that $a_{\mathrm{CP}}$ can vanish when $\theta_{13} \to 0$ as it should.

Large asymmetries are therefore associated with small $\theta_{13}$ and with small oscillation rates. One can then wonder whether having a large asymmetry would not represent a disadvantage from the point of view of statistics. Nevertheless, this is not the case. From (36) we see in fact that, as far as we can set $\Delta m^2_{21} = 0$ in $P_{CP}$, $a_{\mathrm{CP}}/\delta_{\mathrm{stat}} a_{\mathrm{CP}} \propto$

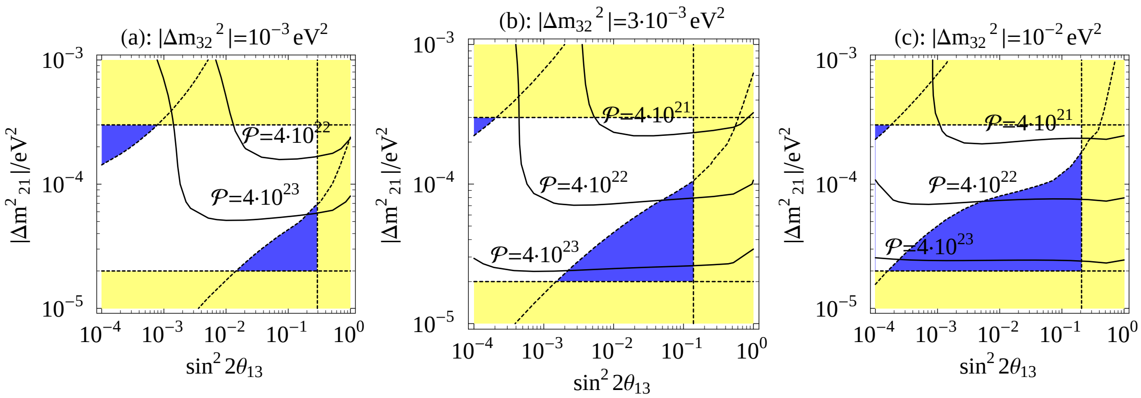


**Figure 4**: *Same as in Fig. 2 for a baseline of $L = 3000$ km.*

$P_{\not{CP}}/\sqrt{P_{CP}}$ is actually independent of $\theta_{13}$. This explains why the sensitivity lines in Figs. 2 are approximately horizontal as far as $\theta_{13}$ is not too small.

### 4.2 Very long baselines

We have seen in the previous Subsection that the possibility of covering a large region of the parameter space with a $L = 732\,\text{km}$ baseline crucially depends on $|\Delta m^2_{32}|$. A better determination of $|\Delta m^2_{32}|$ will come in the next years from the upcoming long-baseline experiments. If $|\Delta m^2_{32}|$ turned out to be close or above the present central value and if the fluxes under discussion could be achieved, the $L = 732\,\text{km}$ baseline could be enough. If, on the contrary, $|\Delta m^2_{32}|$ turned out to be low or the CP-violation size $|\sin^2\delta|$ were small a longer baseline would be necessary. The reason why a longer baseline helps from the point of view of overcoming the statistical error is that the asymmetry grows with $L$, whereas the statistics, and therefore $\delta_{\text{stat}} a_{\text{CP}}$, is approximately independent of $L$. This holds however only as far as $|\Delta_{32}| \ll \pi/2$, or $L \ll 400\,\text{km}\, E_\nu(\,\text{GeV})(3\cdot 10^{-3}\,\text{eV}^2/|\Delta m^2_{32}|)$.

In this Subsection we consider the possibility of a $L = 3000\,\text{km}$ baseline. One can wonder whether having an even longer baseline could not to be better. However, this is not necessarily the case. First of all, $\sin^2\Delta_{32}$ reaches its maximum when $L \sim 400\,\text{km}\, E_\nu(\,\text{GeV})(3\cdot 10^{-3}\,\text{eV}^2/|\Delta m^2_{32}|)$, so that for longer baselines the statistics decreases significantly. Moreover, matter effects grow with $L$ faster than the asymmetry, as pointed out in Section 3.2. This is also confirmed by a comparison of Fig. 2 with Fig. 4, that only differs from Fig. 2 for the length of the baseline, $L = 3000\,\text{km}$ instead of $L = 732\,\text{km}$. We see that the dark-shadowed regions where the matter effect uncertainties are too large are larger in the present $L = 3000\,\text{km}$ case. Therefore, for longer baselines matter effects would represent a major problem. Note, however, that these uncertainties depend on the precision of the future determinations of $|\Delta m^2_{32}|$ and $\theta_{13}$ and they could therefore be smaller than what estimated here (see Section 3.2) .

We see from Fig. 4 that using a $L = 3000\,\text{km}$ baseline and the highest value of $\mathcal{P}$ would allow to cover about 2/3 of the $\Delta m^2_{21}$ parameter space (for $\sin^2 2\theta_{13} \gtrsim 2\cdot 10^{-3}$) in the most unfavourable case $|\Delta m^2_{32}| = 10^{-3}\,\text{eV}^2$ and essentially all of it for larger values of $|\Delta m^2_{32}|$. For such values, the intermediate value of $\mathcal{P}$ would be enough to cover a good

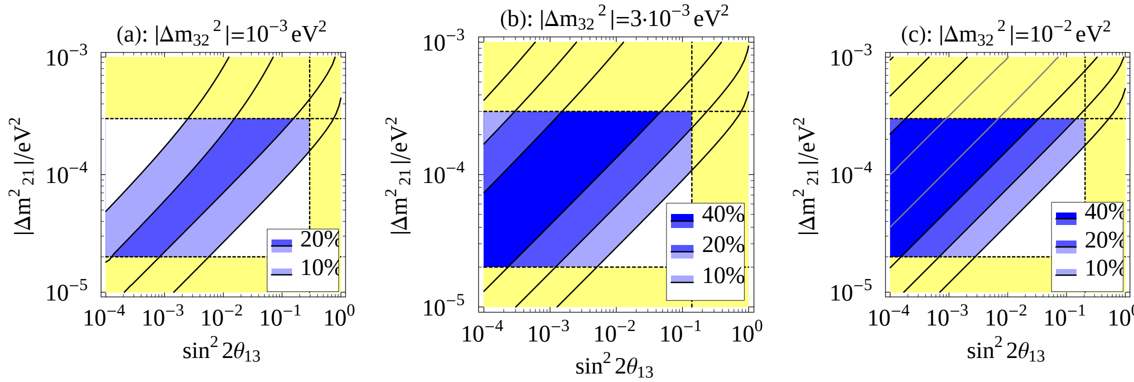


**Figure 5**: *Same as in Fig. 3 for a baseline of* $L = 3000\,km$*.*

half of the $\Delta m^2_{21}$ parameter space.

Finally Fig. 5 shows the contour plot of $|a_{\rm CP}(\delta = \pi/2)| \simeq |a_{\rm CP}/\sin\delta|$ for the $L = 3000\,\rm km$ baseline.

## 5 Discussion and conclusions

The outcomes of the upcoming neutrino experiments will play a crucial role in assessing whether it could be worth making an effort towards a CP-violation measurement. First of all, they will help to understand whether a light sterile neutrino exists and mixes significantly with the active ones. If this were the case, the detection of leptonic CP-violation would be within the capabilities of an intermediate-baseline experiment [1, 5]. If, on the contrary, the standard three neutrino scenario were confirmed, the size and the possibility of measuring CP-violation would crucially depend on the mechanism accounting for the solar neutrino deficit. If the large mixing angle solution were ruled out, there would be no hope of measuring leptonic CP-violation through neutrino oscillations. In the case of the small angle solution, this can be seen using Figs. 2 and 4. In fact, those figures also apply to the small angle solution provided that the parameter $\mathcal{P}$ is interpreted as

$$\mathcal{P} = \frac{N_\mu N_{\rm kT}}{(n_\sigma/5)^2} \cdot \frac{\epsilon_{\mu^\pm}}{30\%} \cdot \sin^2\delta\,\sin^2 2\theta_{12}, \tag{37}$$

where $\theta_{12}$ is now constrained to be within the small angle range. Therefore, in order to get same coverage in the $\sin^2 2\theta_{13}$–$\Delta m^2_{21}$ plane as in the large angle case, it would be necessary to increase $N_\mu \cdot N_{\rm kT}$ by a factor $1/\sin^2 2\theta_{12} \sim 200$.[6] If, on the other hand, the large mixing angle solution were preferred by the data, the measurement of a leptonic CP-violation in the $\nu_e \leftrightarrow \nu_\mu/\bar\nu_e \leftrightarrow \bar\nu_\mu$ channel would represent an exciting experimental challenge.

In the latter case, a precise assessment of the capabilities of a neutrino factory, as well as the determination of the best experimental configuration (essentially the baseline length) would strongly depend on the value of two parameters: the intensity of the muon source

[6]Moreover, that would not still be enough since the $\Delta m^2_{21}$ range of the small angle solution is approximately one order of magnitude lower than the $\Delta m^2_{21}$ range of the large angle solution.

and the precise value of $|\Delta m^2_{32}|$ which will be provided by the long-baseline experiments. Figs. 2 and 4 show what portion of parameter space could be covered for different values of these two parameters and for two possible baselines, $L = 732\,\mathrm{km}$ (Fig. 2) and $L = 3000\,\mathrm{km}$ (Fig. 4). If $|\Delta m^2_{32}|$ turned out to be in the upper part of its present range, $|\Delta m^2_{32}| \sim 10^{-2}\,\mathrm{eV}^2$, and very high-intensity sources could be achieved, the $L = 732\,\mathrm{km}$ baseline could be enough to cover a relevant part of the parameter space. On the other hand, if $|\Delta m^2_{32}|$ turned out to be lower, using an even longer baseline, $L \sim 3000\,\mathrm{km}$, could be necessary. Using baselines longer than 3000 km would require knowing matter effects with a higher precision than that assumed in this paper. Note that, since the relative systematic error due to matter effects grows both with $L$ and $|\Delta m^2_{32}|$ (as opposed to the statistical one, that decreases both with $L$ and $|\Delta m^2_{32}|$) the very long baseline option could turn out to be less appropriate for the higher values of $|\Delta m^2_{32}|$.

Finally, a neutrino factory would give the possibility of measuring $\theta_{13}$ with a high accuracy. This is not only due to the pure and high-intensity neutrino flux, but also due to the possibility of measuring both the $\nu_e \leftrightarrow \nu_\mu$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ channels. A determination of $\sin^2 2\theta_{13}$ made by measuring a single oscillation probability only could in fact be affected by a systematic uncertainty up to 30% associated with CP-violation effects [1] (see also Figs. 3 and 5). On the contrary, measuring both $P_{\nu_e\to\nu_\mu}$ and $P_{\bar{\nu}_e\to\bar{\nu}_\mu}$ would allow to get rid of the CP-violating part of the probability $P_{\not{CP}}$ by computing $(P_{\nu_e\to\nu_\mu}+P_{\bar{\nu}_e\to\bar{\nu}_\mu})/2 = P_{CP}$ and to get rid of the further dependence of $P_{CP}$ on $\delta$ by computing the CP-asymmetry $a_{\mathrm{CP}}$.

## Acknowledgments

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## References

[1] K. Dick, M. Freund, M. Lindner and A. Romanino, (1999), hep-ph/9903308.

[2] S. H. Geer, Phys. Rev. **D57**, 6989 (1998), hep-ph/9712290.

[3] A. D. Rujula, M. B. Gavela and P. Hernandez, Nucl. Phys. **B547**, 21 (1999), hep-ph/9811390.

[4] V. Barger, S. Geer and K. Whisnant, (1999), hep-ph/9906487.

[5] A. Donini, M. B. Gavela, P. Hernandez and S. Rigolin, (1999), hep-ph/9909254.

[6] V. Barger, Y.-B. Dai, K. Whisnant and B.-L. Young, Phys. Rev. **D59**, 113010 (1999), hep-ph/9901388.

[7] A. Kalliomaki, J. Maalampi and M. Tanimoto, (1999), hep-ph/9909301.

[8] SuperKamiokande, Y. Fukuda *et al.*, Phys. Rev. Lett. **82**, 1810 (1999), hep-ex/9812009.

[9] M. C. Gonzalez-Garcia, P. C. de Holanda, C. Pena-Garay and J. W. F. Valle, (1999), hep-ph/9906469.

[10] SuperKamiokande, Y. Fukuda *et al.*, (1999), hep-ex/9908049.

[11] V. Barger, T. J. Weiler and K. Whisnant, Phys. Lett. **B440**, 1 (1998), hep-ph/9807319;
G. L. Fogli, E. Lisi, A. Marrone and G. Scioscia, Phys. Rev. **D59**, 033001 (1999), hep-ph/9808205.

[12] CHOOZ, M. Apollonio *et al.*, Phys. Lett. **B420**, 397 (1998), hep-ex/9711002,

[13] J. Arafune, M. Koike and J. Sato, Phys. Rev. **D56**, 3093 (1997), hep-ph/9703351;
H. Minakata and H. Nunokawa, Phys. Lett. **B413**, 369 (1997), hep-ph/9706281;
M. Tanimoto, (1999), hep-ph/9906516.

[14] A. Cervera, F. Dydak and J. Gómez-Cadenas, Nufact '99 Workshop, July 5–9th, Lyon.

[15] Nufact '99 Workshop, July 5–9th, Lyon. See e.g. the Summary of Detector/Neutrino Beam Parameters by D. Harris, available at http://lyopsr.in2p3.fr/nufact99/talks/harrisconc.ps.gz.

[16] R. Barbieri, L. J. Hall, D. Smith, A. Strumia and N. Weiner, JHEP **12**, 017 (1998), hep-ph/9807235.